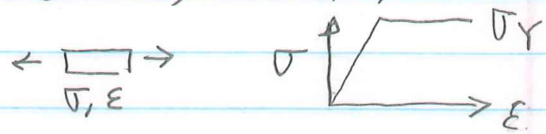
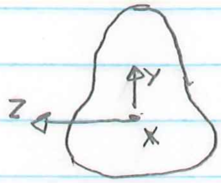
17. Ultimate value of bending moment
(symmetric cross-section, homogeneous, isotropic, elastic-plastic material)

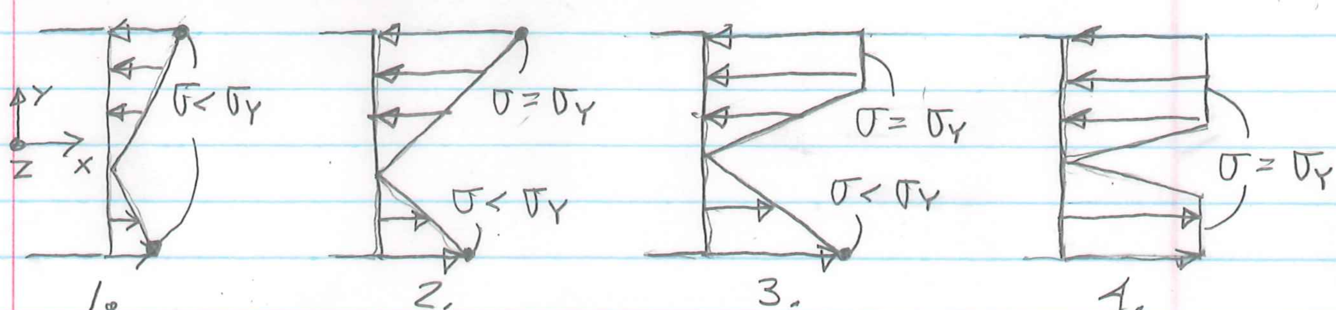




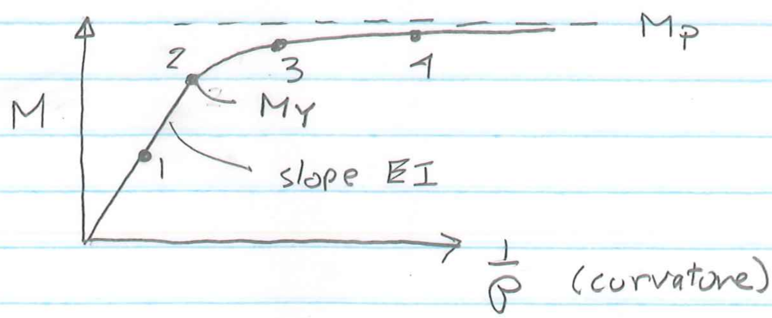
x-y is the plane of symmetry. M applied about the z axis.

We still have plane sections remaining plane and existence of a neutral axis where $\varepsilon_x = 0$; however, after yielding occurs, the neutral axis may no longer be a centroidal axis. Still have $\varepsilon_x = -y/\rho$ and only the σ_x stress component.

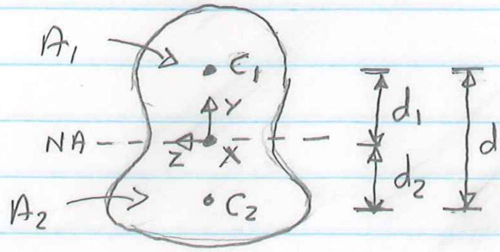
Examine the σ_x distribution on a cross-section as M increases (states 1, 2, 3 and 4). Denotes stresses by σ and σ_Y.



State 2 is when a fiber first reaches yield (at the top), so M = the yield moment M_Y. In state 4, the entire cross-section is approching yield, so $M \to$ the plastic moment M_P.



As $M \to M_P$, the location of the neutral axis approaches the line that divides the beam into equal areas., since the resultant axial force on a cross-section equals zero.



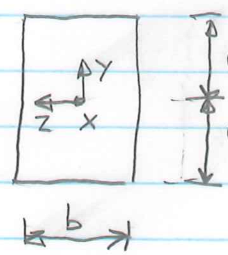
1 and 2 denote compression and tension zones. C_1 and C_2 are centroids of these zones.

The value of M_P is computed as $M_P = \sigma_Y A_1 d_1 + \sigma_Y A_2 d_2$
$= \sigma_Y A d/2$

since $A_1 = A_2 = A/2$.

This can be written as $M_P = \sigma_Y Z$, where Z is the plastic section modulus $= Ad/2$. Recall $M_Y = \sigma_Y S$ where S is the section modulus. Therefore, $M_P/M_Y = Z/S$

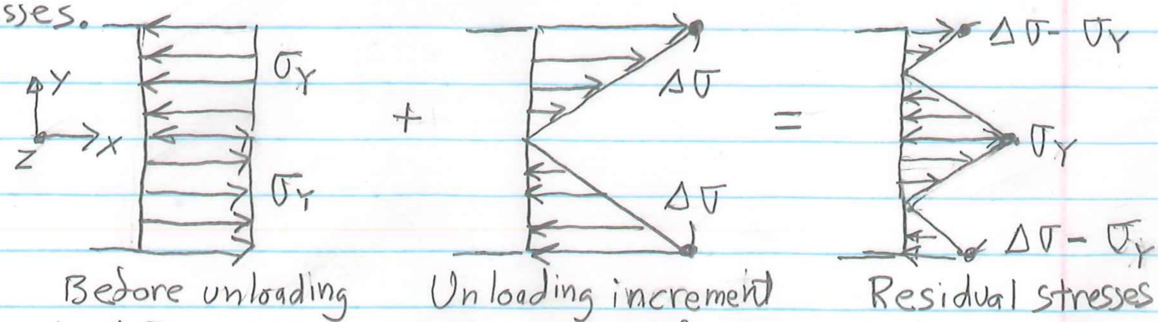
Example: Rectangular cross-section



$$Z = 2cb \cdot \frac{c}{2} = bc^2$$
$$S = I/c = = \frac{1}{12}(2c)^3 b/c =$$
$$= \frac{2}{3} bc^2$$

Thus, $\frac{M_P}{M_Y} = \frac{3}{2}$

Example: A beam with rectangular cross-section is loaded to M_P then unloaded. Find the distribution of residual stresses.



Before unloading Unloading increment Residual stresses

To find $\Delta\sigma$, use $M = 0$ on the residual stress distribution. Get $\Delta\sigma = \frac{3}{2}\sigma_Y$.